Prove: A square's digits

A positive integer consists only of some arrangement of the digits 0, 3, 5, 7 and 8 (each possibly used multiple times). Show that it can never be a perfect square.

The South
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Game: Catch Up

Catch up is a two-player adding game where players gather tokens competing for the highest total sum.

The game is played with **n** tokens numbered **1** to **n.** To start the game, Player A takes a single token. Player B then takes tokens one at a time, adding their values up until their tokens' total sum is the same or higher than Player A's, which ends Player B's turn. Player A now does the same, taking tokens until their total is at least as much as Player B's. Play alternates until all of the tokens have been used up.

Example for **n** = 6:
A takes #3. (A=3)
B takes #6. (B=6)
A takes #2, (A=5) then #5 (A=10)
B takes #1 (B=7) then #4 (B=11)
With all tokens taken, A has a score of 10 and B has a score of 11, so B wins.

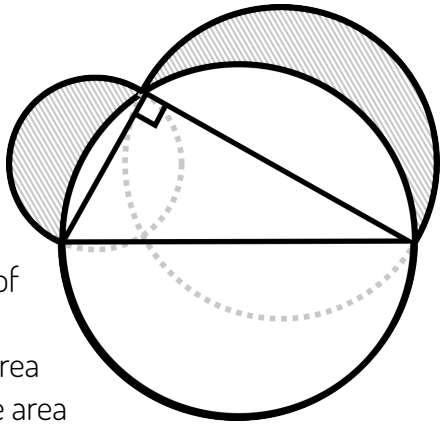
Problems:
For what values of **n** can Player A force a win? For what values of **n** can Player B force a win? Are there values where optimal play will force a draw? Over all **n**, does the game favour a particular player?

Where are the queens?

Source: robeastaway.com/blog/black-queen

You are drawing cards from a standard, well-shuffled deck of 52 cards, trying to determine how many cards you have to draw on average before you draw a queen, when you are approached by a mysterious OMG attendee you're sure you've never met before. She takes the shuffled deck, and tells you she will reveal all 52 cards, one at a time, but first she wants you to predict the position in the pack of the **second black queen**. If your guess is correct, she will reward you with an exciting mathematical prize.
Even though your odds aren't great, you want to make your chances as high as possible. Which position in the pack should you choose?

Solve: Lunes of Hippocrates



In figure 1 (right) each of the three sides of the right triangle is the diameter of one of the three circles.
Can you work out the shaded area of the two lunes in terms of the area of the right triangle?

figure 1

Puzzle: sixes and sevens with you

If you write out all the whole numbers from 0 to 9999, how many numbers would you have in which both the digits 6 and 7 are present?

Solve for x: $x^x = 4^{(x+16)}$

Puzzle: The average age in the room

Twins Niamh and Saoirse arrive late to a meeting. When Niamh enters the room, the mean age of everyone in the room increases by four years. She is immediately followed by Saoirse, and the mean age increases by a further three. How many people are now at the meeting?